Technical Comments

Comments on "Method-of-Characteristics Solution of a Rarefied, Monatomic, Gaseous Jet Expansion into a Vacuum"

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THE purpose of this comment is firstly, to point out an apparent misconception in the application of the so-called hypersonic approximation of rarefied gas dynamics in a recent paper by Robertson and Willis,[1] and secondly to observe that when this misconception is removed the problem reduces to that of the hypersonic freejet treated by the present author.[2]

The hypersonic approximation of rarefied gas dynamics involves the neglect of the highest order moments occurring in each moment equation. In the paper of Robertson and Willis this procedure is not adhered to and consequently their truncated set of equations is not consistent. They neglect higher order moments in the second order moment equations, i.e., heat flux terms, but inconsistently retain the stress terms in the momentum equations. If the correct procedure is adopted and the stress terms p_t, p_n, p_s, p_b in the momentum equations [Eqs. (4) and (5) of Robertson and Willis's paper] are neglected, Eq. (5) becomes $\partial\theta/\partial s = 0$, implying straight streamlines, while Eq. (4) yields $\partial u/\partial s = 0$, and the velocity is constant along each streamline. It is now a simple matter to show that $\rho \sim 1/r^2$ along each streamline. On the other hand, if one wishes to retain the stress components in the momentum equations, as Robertson and Willis do, then there is no mathematical or indeed physical basis for a truncation procedure at all. Although it is possible to compare their procedure to the infinite Prandtl Number approximation of continuum theories, it is difficult to justify calling this a hypersonic approximation.

The hypersonic approximation was originally introduced by Hamel and Willis[3] and subsequently developed as a systematic approximation procedure by Freeman[4] for the steady spherically symmetric expansion. It was applied to the free jet problem[2] using the method of matched asymptotic expansions; the salient conclusions of this work are as follows:

a) In the outer region of the freejet where the hypersonic approximation can be made, the streamlines are straight and radially directed. In addition the velocity is constant along each line.

b) The density varies as $1/r^2$ along each streamline (r is a suitable radial coordinate).

c) The radial temperature freezes along each streamline but the final frozen temperature varies with angular distance from the centerline of the jet. The transverse temperatures go to zero. Indeed it was pointed out that along each streamline the expansion could be regarded as a spherical expansion provided the radial coordinate and flow variables were appropriately scaled along each line. These are basically the conclusions drawn by Robertson and Willis in their paper.

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References

[1] Robertson, S. J. and Willis, D. R., "Method-of-Characteristics Solution of a Rarefied, Monatomic Gaseous Jet Expansion into a Vacuum," *AIAA Journal*, Vol. 9, No. 2, Feb. 1971, pp. 291–296.

[2] Grundy, R. E., "Axially Symmetric Expansion of a Monatomic Gas from an Orifice into a Vacuum," *The Physics of Fluids*, Vol. 12, No. 10, Oct. 1969, pp. 2011–2018.

[3] Hamel, B. B. and Willis, D. R., "Kinetic Theory of Source Flow Expansion with Application to the Free Jet," *The Physics of Fluids*, Vol. 9, No. 5, May 1966, pp. 829–841.

[4] Freeman, N. C., "Solution of the Boltzmznn Equation for Expanding Flows," *AIAA Journal*, Vol. 5, No. 9, Sept. 1967, pp. 1696–1698.

Reply by Authors to R. E. Grundy

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THE substance of Grundy's comment is that we were not justified in dropping the heat-transfer terms in the second order moment equations while retaining the stress terms (pressure and shear) in the momentum equations.

From a purely mathematical standpoint, Grundy is correct in his criticism of our use of the hypersonic approximation in this fashion. The stress terms are required, however, if one is to attempt an analysis of the change in flow pattern from a parallel stream at the nozzle exit to a source-like flow downstream. If the heat-transfer terms are not dropped, then they must be approximated. We chose to ignore them since they would become negligible anyway as the expansion proceeds downstream.

Grundy is correct in noting that our results basically concur with his conclusions concerning the suitability of matching the inviscid solution to the asymptotic solution along each streamline. Grundy's results appeared in the open literature after our results were first documented in the form of a technical report (Ref. 6 of our paper). We regret not giving recognition to his results when we re-wrote our results for publication in the *AIAA Journal*.

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